# Worked Example 2 (Version 1)

# Design of concrete cantilever retaining walls to resist earthquake loading for residential sites

Worked example to accompany MBIE Guidance on the seismic design of retaining structures for residential sites in Greater Christchurch (Version 2) November 2014

## Introduction

Cantilever concrete retaining walls are commonly used for residential purposes, often as integral basement walls. Usually the cantilever wall stem is of concrete block construction rising from an in-situ concrete foundation.

The following worked example is for a free-standing cantilever wall that is considered sufficiently flexible for active soil pressures to be used for design. Where used as integral basement walls they are often buttressed by return walls and floor diaphragms which may make them too stiff for active soil pressures to develop requiring higher design loads and a different design approach.

### 1.1 Possible modes of failure

Possible modes of failure for free-standing concrete cantilever retaining walls are illustrated in cartoon fashion in Figure X.1. A complete design should address each of these modes of failure where appropriate.

a) **Wall stem structural failure:** The wall stem fails in bending. Most likely location is at the base of the wall where the stem connects to the foundation.

b) **Foundation bearing failure:** A bearing failure of the soil under the toe of the foundation and a forwards rotation of the wall.

c) **Sliding failure of wall:** Possible mode for non-cohesive soils. Wall moves outwards with passive failure of soil in front of foundation and active failure of soil behind wall. Often a key is required beneath the foundation to prevent sliding.

d) **Deep seated rotational failure:** Possible mode for cohesive soils. Factor of safety controlled by increasing length of heel or depth of key. Factor of safety calculated using limiting equilibrium “Bishop” analysis or similar. Unlikely to govern design unless wall is embedded into sloping ground with sloping backfill or there is a weak layer at the toe of the wall.

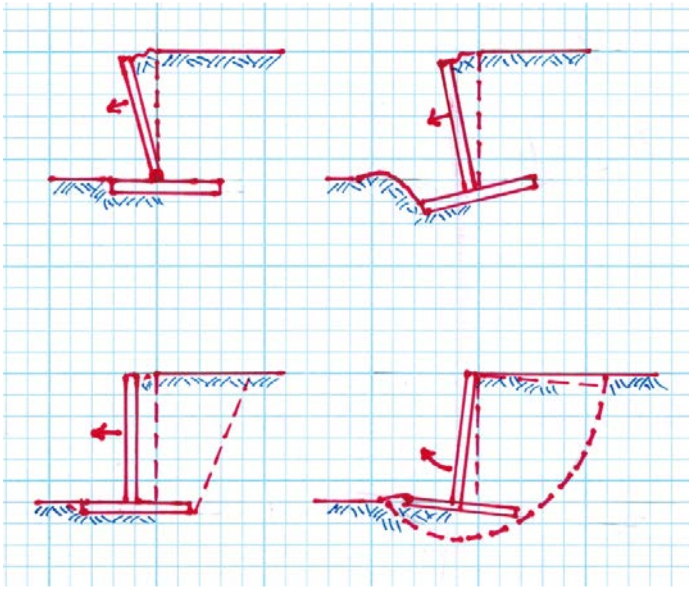


Figure X.1. Possible modes of failure for free-standing concrete cantilever retaining walls.

The following worked example uses a simplified LRFD design procedure with load and resistance factors taken from B1/VM4. It is considered suitable for common residential situations with competent soils.

This procedure is intended to be readily calculated by hand, although use of calculation software such as Mathcad or Excel will be useful for design iterations. The example calculations are made here using Mathcad.

### 1.2 Example Wall

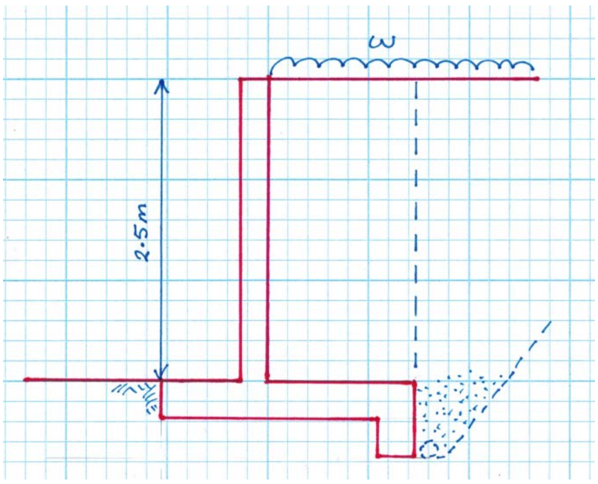



Figure X.2. Concrete cantilever wall example.

The example wall is shown in Figure X.2. The wall is assumed to be located in the Christchurch Port Hills. The following design assumptions were made:

<u>Soil type:</u> Port Hills loess

<u>Strength parameters:</u> c = 0, $\phi$ = 30 degrees

Drained strength parameters for Port Hills loess were assumed for the long term, gravity only load case. For the earthquake load case, the foundations in loess were designed assuming undrained strength, c = 50 KN/m$^2$, $\phi$ = 0 degrees. (Following the recommendations given in the Guidelines).

Wall situation: Case 3: Retaining wall downslope and supporting dwelling foundations

Surcharge: The surcharge from the dwelling was assumed to be 5 kN/m$^2$ averaged across the active soil wedge for the gravity case and 4 kN/m$^2$ for the earthquake case. Surcharge should be calculated using:

$\omega$ = 1.2 G + 0.4 Q for the gravity case

$\omega$ = G + 0.3 Q for the earthquake case.

Seismic parameters:

$C(T) = C_h(T)ZRN(T,D)$ Equation (1.1) from Guidelines

$C_h(T)$ 1.33 for Class C assuming shallow soil site

Z = 0.3 for Christchurch for ULS

R = Return period factor = 1.0 for Importance Level 2 walls, ULS

N(T,D) = Near fault factor which may be taken = 1.0 for residential retaining walls

*C(T)* = 0.3 x 1.33 = 0.4

$C(T,A_{topo}) = C(T)A_{topo}$ Equation (1.2) from Guidelines

$A_{topo}$ = 1.0 assuming site is not near cliff edge or ridge top

$C(T,A_{topo})$ = 0.4 x 1.0 = 0.4

$k_h = C(T,A_{topo})W_d$ Equation (1.3) from Guidelines

$W_d$ = wall displacement factor, given in Table 2 from Guidelines as 0.5 (refer to Table 1 for wall case, then Table 2 for $W_d$)

$k_h$ = 0.4 x 0.5 = 0.2

Note that by adopting $W_d$ = 0.5 it is implicitly assumed that the wall and the retained ground are likely to yield and accumulate permanent displacement during the design earthquake. Wall elements must be sufficiently resilient and/or ductile to accommodate the displacement. Some settlement of retained material behind the wall should also be expected following an earthquake.

## Step 1. Initial trial geometry

The main variables for geometry are the length of the toe, the length of the heel, and the depth of the key. These will be refined during the analysis below. The thickness of the wall stem and footing should be refined during the structural design process. The optimum location for the key is at the end of the heel, as shown in Figure X.2. The analytical model used for the design is illustrated in Figure X.3.

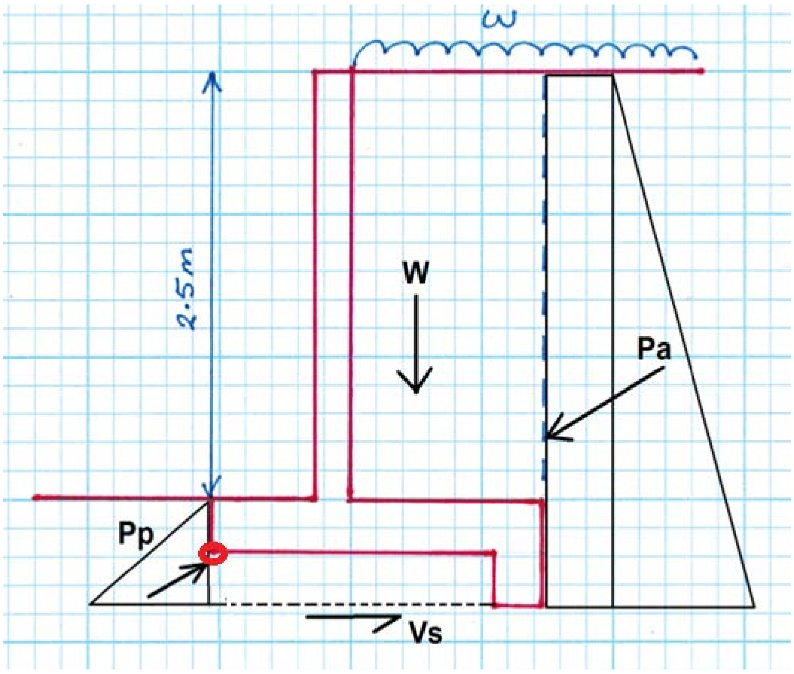


Figure X.3. Analytical model used for gravity design of free-standing concrete cantilever wall (moments taken about point O).

## Step 2. Foundation bearing (gravity case)

The foundation bearing capacity (gravity case) will usually govern the design of the wall dimensions and is checked first. The soil under the toe of the foundation in particular is working very hard to resist the vertical bearing loads, sliding shear, and to provide passive resistance to sliding.

For the following simplified procedure, the "middle third rule" is applied, whereby the wall foundation is dimensioned so that the resultant vertical force acts through the "middle third" of the footing. If the "middle third rule" is not applied, then a more rigorous analysis of the bearing capacity of the wall foundation should be undertaken.

The bearing capacity of the foundation ***must*** be calculated taking into account the effect of simultaneous horizontal loads applied to the foundation from the soil pressure (i.e. by applying load inclination factors), and using the reduced, effective width of the foundation from the eccentricity of the resultant vertical load. Where there is confidence in the properties of the soil backfill in front of the toe of the footing, then the net horizontal load considered when calculating the load inclination factors for the bearing capacity may be reduced by the passive soil force acting against the footing (refer to Brinch-Hansen 1970), in which case the depth factors must be set to 1.0 (i.e. the shear strength of the soil above the founding depth of the footing cannot be counted twice).

In the worked example, the passive soil resistance has been neglected (conservatively) when calculating the load inclination factors and bearing capacity, as follows.

**Conc Cantilever Wall Parameters**

| | |
|---|---|
| $H_w := 2.5 \cdot m$ | Height of wall |
| $L_{stem} := 0.2 \cdot m$ | Thickness of wall stem |
| $L_{toe} := 0.65 \cdot m$ | Length of toe |
| $L_{base} := 0.25 \cdot m$ | Thickness of base |
| $L_{heel} := 1.0 \cdot m$ | Length of heel |
| $L_{key} := 0.2 \cdot m$ | Depth of shear key |
| $\phi := 30 \cdot deg$ | Soil friction angle |
| $\gamma := 18 \cdot \frac{kN}{m^3}$ | Soil unit weight |
| $\gamma_{conc} := 24.5 \cdot \frac{kN}{m^3}$ | Concrete unit weight |
| $\omega_g := 5 \cdot \frac{kN}{m^2}$ | Factored surcharge, gravity case, destabilising (1.2 G+ 0.4 Q) |
| $\omega_{gs} := 3 \cdot \frac{kN}{m^2}$ | Factored surcharge, gravity case, stabilising (0.9 G) |

**LRFD parameters**

| | |
|---|---|
| $\Phi_{bc} := 0.5$ | Resistance factor for bearing capacity, gravity case |
| $\Phi_{sl} := 0.8$ | Resistance factor for sliding, gravity case |
| $\Phi_p := 0.5$ | Resistance factor for passive earth pressure, gravity case |
| $\alpha_{G\_stab} := 0.9$ | Load factor for self-weight (stabilising) |
| $\alpha_{G\_destab} := 1.2$ | Load factor for self-weight (de-stabilising) |
| $\alpha_{EP\_static} := 1.5$ | Load factor for earth pressure, gravity case (de-stabilising) |

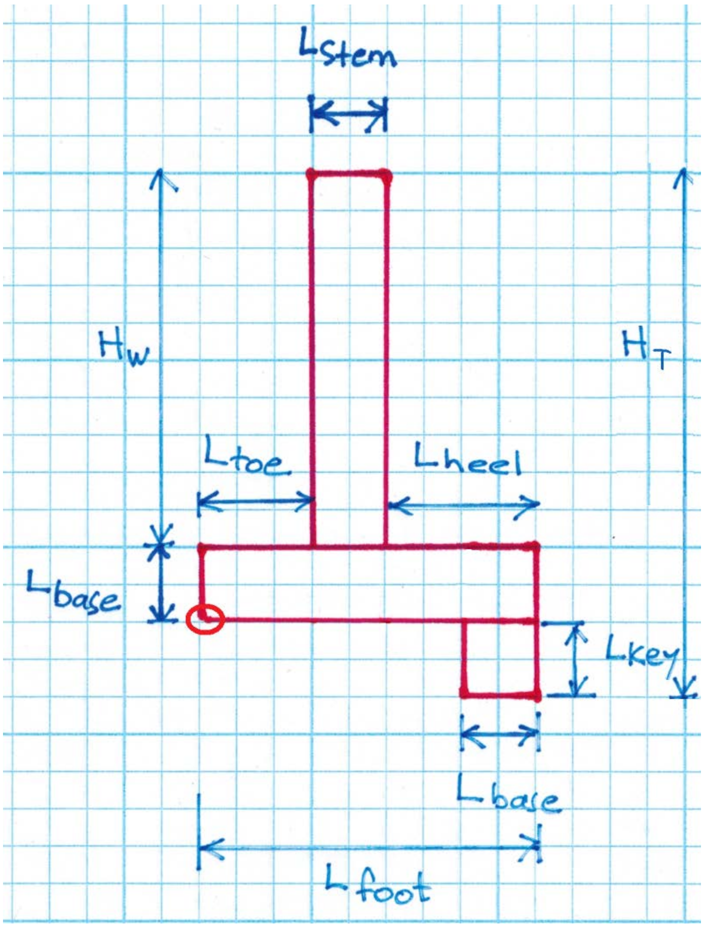



Figure X.4. Parameter definition.

**Computed parameters**

| | | |
|---|---|---|
| $L_{foot} := L_{toe} + L_{stem} + L_{heel}$ | $L_{foot} = 1.85\,m$ | Width of footing |
| $H_T := H_w + L_{base} + L_{key}$ | $H_T = 2.95\,m$ | Total height of structure |
| $W_{foot} := L_{foot} \cdot L_{base} \cdot \gamma_{conc}$ | $W_{foot} = 11.331 \cdot \frac{kN}{m}$ | Weight of footing |
| $W_{key} := L_{key} \cdot L_{base} \cdot \gamma_{conc}$ | $W_{key} = 1.225 \cdot \frac{kN}{m}$ | Weight of key (same thickness as base) |
| $W_{stem} := H_w \cdot L_{stem} \cdot \gamma_{conc}$ | $W_{stem} = 12.25 \cdot \frac{kN}{m}$ | Weight of wall stem |
| $W_{soil} := L_{heel} \cdot H_w \cdot \gamma$ | $W_{soil} = 45 \cdot \frac{kN}{m}$ | Weight of soil above heel |

$K_a := 0.3$ From MO equations (Coulomb) $\phi$ = 30 degrees, $\delta = \phi$, i = 0 degrees

$K_p := 5.5$ From NAVFAC DM7 $\phi$ = 30 degrees, $\delta$ = 2/3 $\phi$

(Note: A chart giving values of $K_a$ and $K_p$ based on the log-spiral solutions of Caquot and Kerisel is appended to this example).

**Check "middle third rule"**
Factored moments about toe, divided by factored vertical forces neglecting passive resistance, which may not be mobilised.

$\delta_a := \phi$ — Interface friction angle, wall virtual back face

$P_a := 0.5 \cdot K_a \cdot \gamma \cdot H_T^2$ $\quad P_a = 23.497 \cdot \frac{kN}{m}$ — Active thrust, soil weight component

$P_{av} := P_a \cdot \sin(\delta_a)$ $\quad P_{ah} := P_a \cdot \cos(\delta_a)$ — Vertical, horizontal components

$P_{a\omega} := \omega_g \cdot K_a \cdot H_T$ $\quad P_{a\omega} = 4.425 \cdot \frac{kN}{m}$ — Active thrust, surcharge component

$P_{av\omega} := P_{a\omega} \cdot \sin(\delta_a)$ $\quad P_{ah\omega} := P_{a\omega} \cdot \cos(\delta_a)$ — Vertical, horizontal components

$P_\omega := \omega_{gs} \cdot L_{heel}$ $\quad P_\omega = 3 \cdot \frac{kN}{m}$ — Surcharge above heel

$M_{ah} := \left[P_{ah} \cdot \left(\frac{H_T}{3} - L_{key}\right) + P_{ah\omega} \cdot \left(\frac{H_T}{2} - L_{key}\right)\right] \cdot \alpha_{EP\_static}$ — Moment from horizontal active pressure (+ve)

$M_{av} := (P_{av} + P_{av\omega}) \cdot L_{foot}$ — Moment from vertical active pressure (-ve)

$M_\omega := P_\omega \cdot \left(L_{foot} - \frac{L_{heel}}{2}\right)$ — Moment from surcharge above heel (-ve)

$$M_G := \left[W_{foot} \cdot \frac{L_{foot}}{2} + W_{stem} \cdot \left(L_{toe} + \frac{L_{stem}}{2}\right) + W_{key} \cdot \left(L_{foot} - \frac{L_{key}}{2}\right) + W_{soil} \cdot \left(L_{foot} - \frac{L_{heel}}{2}\right)\right] \cdot \alpha_{G\_stab}$$

^ Restoring moment from self weight of wall and soil above heel (-ve)

$M_{ah} = 31.239 \cdot \frac{kN \cdot m}{m}$ $\quad M_{av} = 25.828 \cdot \frac{kN \cdot m}{m}$ $\quad M_G = 74.306 \cdot \frac{kN \cdot m}{m}$ $\quad M_\omega = 4.05 \cdot kN \cdot \frac{m}{m}$

$M_{net} := M_{ah} - M_{av} - M_G - M_\omega$ — Net moment about toe, neglecting passive thrust

$M_{net} = -72.945 \cdot kN \cdot \frac{m}{m}$ — Net moment must be < 0 for stability

$$P_{vert} := (W_{foot} + W_{stem} + W_{key} + W_{soil}) \cdot \alpha_{G\_stab} + P_{av} + P_{av\omega} + P_\omega$$

$P_{vert} = 79.787 \cdot \frac{kN}{m}$ — Factored vertical load on footing

$L_{net} := \frac{-M_{net}}{P_{vert}}$ $\quad L_{net} = 0.914 m$ — Line of action of net vertical force (distance from toe)

$L_{third} := \frac{1}{3} \cdot L_{foot}$ $\quad L_{third} = 0.617 m$ $\quad 2 \cdot L_{third} = 1.233 m$

Adjust wall proportions until line of action is within "middle third"

Note that the vertical component of active thrust is not factored (i.e. $\alpha$ = 1). The horizontal component of active thrust is factored ($\alpha$ = 1.5) to account for uncertainty of soil properties. But, uncertainty in soil properties does not significantly affect the vertical component which will remain about the same even if the actual soil friction angle is less than assumed.

The self-weight components are here factored ***down*** ($\alpha$ = 0.9) to account for uncertainty because they are "stabilising" in this context, even though contributing to the vertical load on the footing.

**Check bearing capacity**
The "effective" width of the footing must be established, together with the net horizontal and vertical loads acting on the footing:

$B_{eff} := 2 \cdot L_{net}$ $\quad B_{eff} = 1.829 m < L_{foot} = 1.85 m$ — Effective footing width (less than total width)

$V_u := P_{vert}$ $\quad V_u = 79.787 \cdot \frac{kN}{m}$ — Ultimate vertical load on footing

$H_u := (P_{ah} + P_{ah\omega}) \cdot \alpha_{EP\_static}$ $\quad H_u = 36.271 \cdot \frac{kN}{m}$ — Ultimate hozrizontal load on footing

Detailed bearing capacity calculations are appended, and give the following result:

$$q_u = 91.968 \cdot \frac{kN}{m^2}$$

$$V_{ustar} := B_{eff} \cdot q_u \cdot \Phi_{bc} \qquad V_{ustar} = 84.082 \cdot \frac{kN}{m} \quad > \quad V_u = 79.787 \cdot \frac{kN}{m}$$

$V_{star} > V_u$ therefore bearing capacity OK for gravity case.

## Step 3. Wall sliding (gravity case)

The sliding analysis is carried out with reference to the model shown in Figure X.3. The weight of the block of soil underneath the footing and mobilised by the key is included in the calculation of base friction, $V_s$. All of the self-weight components are here factored ***down*** ($\alpha$ = 0.9) to account for uncertainty because they are "stabilising" in this context.

The vertical component of active thrust is not factored (i.e. $\alpha$ = 1), as before. The vertical component of passive resistance is also not factored (i.e. $\alpha$ = 1) because it is "de-stabilising" in this context.

**Check wall sliding on base**

| | | |
|---|---|---|
| $W_{slide} := (L_{foot} - L_{base}) \cdot L_{key} \cdot \gamma$ | $W_{slide} = 5.76 \cdot \frac{kN}{m}$ | Weight of soil trapped under footing |
| $\delta_p := \frac{2}{3} \cdot \phi$ | | Interface friction angle for passive pressure |
| $P_p := 0.5 \cdot K_p \cdot \gamma \cdot (L_{base} + L_{key})^2$ | $P_p = 10.024 \cdot \frac{kN}{m}$ | Passive resistance |
| $P_{ph} := P_p \cdot \cos(\delta_p) \qquad P_{pv} := P_p \cdot \sin(\delta_p)$ | | Horizontal, vertical components |
| $H_s := (V_u + W_{slide} \cdot \alpha_{G\_stab} - P_{pv}) \cdot \tan(\phi)$ | $H_s = 47.078 \cdot \frac{kN}{m}$ | Friction under footing |
| $H_{star} := P_{ph} \cdot \Phi_p + H_s \cdot \Phi_{sl}$ | | Factored ultimate resistance |

$$H_{star} = 42.372 \cdot \frac{kN}{m} \qquad H_u = 36.271 \cdot \frac{kN}{m}$$

Factored resistance > factored load therefore OK.

## Step 4. Wall stem bending strength (gravity case)

The wall stem may fail in bending. The maximum bending moment will be at the base of the stem and may be calculated using the analytical model shown in Figure X.5. The surcharge above the heel is included as a worst case. The calculation of the bending strength of the wall should be carried out in accordance with the relevant material code.

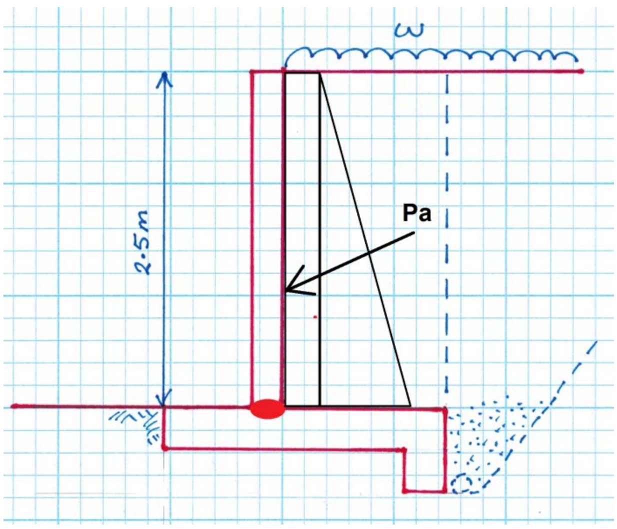



Figure X.5. Analytical model for calculating bending moment in wall stem

**Calculate maximum bending moment in wall stem**

Assume that wall has water proof membrane with padding i.e. negligible interface friction

$\delta_s := 0$

$K_{as} := 0.33$ From MO equations (Coulomb) $\phi$ = 30 degrees, $\delta$ = 0, i = 0

$P_{as} := 0.5 \cdot K_{as} \cdot \gamma \cdot H_w^2$ $P_{as} = 18.563 \cdot \frac{kN}{m}$ Active thrust

$P_{ahs} := P_{as} \cdot \cos(\delta_s)$ Horizontal component

$P_{a\omega s} := \omega_g \cdot K_{as} \cdot H_w$ $P_{a\omega s} = 4.125 \cdot \frac{kN}{m}$ Active thrust, surcharge component

$P_{ah\omega s} := P_{a\omega s} \cdot \cos(\delta_s)$ Horizontal component

$M_u := \left( P_{ahs} \cdot \frac{H_w}{3} + P_{ah\omega s} \cdot \frac{H_w}{2} \right) \cdot \alpha_{EP\_static}$ Ultimate bending moment in stem

$M_u = 30.938 \cdot \frac{kN \cdot m}{m}$

The bending capacity of the wall stem under action $M_u$ needs to be checked using the relevant material code.

## Step 5. Foundation bearing (earthquake case)

The foundation bearing capacity is checked for the earthquake case using the same geometry developed for the gravity case and including the earthquake inertia loads from the self-weight of the wall and from the soil above the heel according to the analytical model shown in Figure X.6.

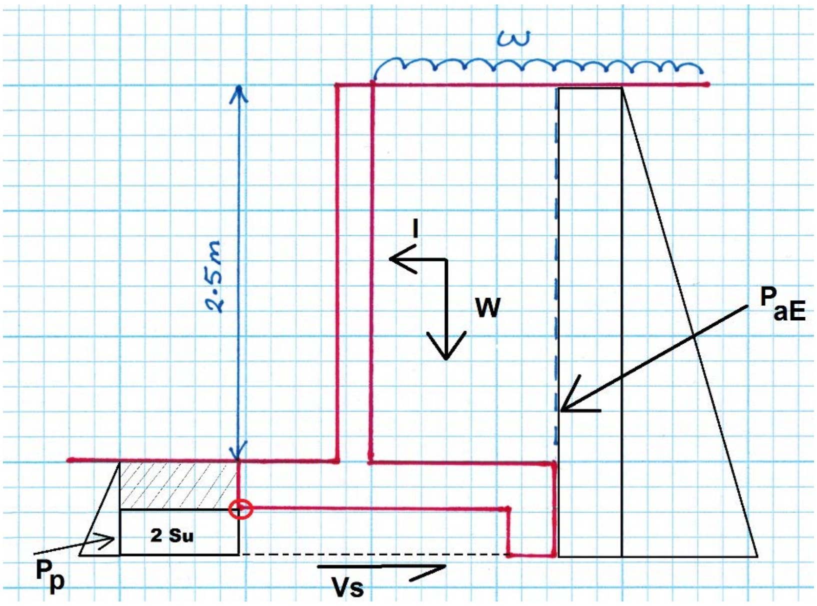



Figure X.6. Analytical model for earthquake case.

For the earthquake case, the undrained shear strength of the foundation soil may be assumed for Port Hills loess when calculating the passive soil resistance. For the example, $S_u$ =50 KN/m$^2$ was assumed. The passive soil distribution is shown in Figure X.6 with the cohesive contribution = 2 c where c = $S_u$ and $K_p$ = 1 for $\phi$ = 0.

Where the ground surface immediately in front of the wall is exposed, the passive resistance may be ineffective near to the ground surface because of desiccation and cracking and disturbance during excavation of the footing. For the example, the cohesive component of passive resistance was neglected down to the base of the concrete footing. For other situations where the ground surface is protected by pavement it may be appropriate to include the cohesive component of passive soil resistance over the full depth of embedment, using judgement.

Using the same simplified procedure as for the gravity case, the "middle third rule" is again checked.

The bearing capacity of the foundation, again, ***must*** be calculated taking into account the effect of simultaneous horizontal loads applied to the foundation from the soil pressure (i.e. by applying load inclination factors), and using the reduced, effective width of the foundation from the eccentricity of the resultant vertical load. For the earthquake case, the LRFD parameters are all set to unity, as discussed in the guidelines, assuming that the loess foundation soil will not be subject to strength loss during earthquake shaking or strain softening as a result of soil yielding.

$\omega_{eq} := 4 \cdot \frac{kN}{m^2}$ Factored surcharge, EQ, destabilising (G + Eu + 0.3Q)

$\omega_{gs} := 3 \cdot \frac{kN}{m^2}$ Factored surcharge, stabilising (0.9 G)

$k_h := 0.2$ Seismic coefficient

**LRFD parameters**

All set to 1.0

$K_{aE} := 0.471$ From M-O equations kh = 0.2, $\phi$ = 30 degrees, $\delta = \phi$, i = 0 degrees

**Check "middle third rule"**

Factored moments about rotation point, divided by factored vertical forces neglecting passive resistance, which may not be mobilised.

$\delta_a := \phi$ Interface friction angle, wall virtual back face

$P_a := 0.5 \cdot K_{aE} \cdot \gamma \cdot H_T^2$ $P_a = 36.89 \cdot \frac{kN}{m}$ Active thrust, soil weight component

$P_{av} := P_a \cdot \sin(\delta_a)$ $P_{ah} := P_a \cdot \cos(\delta_a)$ Vertical, horizontal components

$P_{a\omega} := \omega_{eq} \cdot K_{aE} \cdot H_T$ $P_{a\omega} = 5.558 \cdot \frac{kN}{m}$ Active thrust, surcharge component

$P_{av\omega} := P_{a\omega} \cdot \sin(\delta_a)$ $P_{ah\omega} := P_{a\omega} \cdot \cos(\delta_a)$ Vertical, horizontal components

$P_\omega := \omega_{gs} \cdot L_{heel}$ $P_\omega = 3 \cdot \frac{kN}{m}$ Surcharge above heel

The inertia of the wall structural elements and soil located above the heel (treated as part of the wall) are added, as follows:

$I_{foot} := W_{foot} \cdot k_h$ $I_{key} := W_{key} \cdot k_h$ $I_{stem} := W_{stem} \cdot k_h$ $I_{soil} := W_{soil} \cdot k_h$ Inertia forces, structure

$M_{ah} := \left[P_{ah} \cdot \left(\frac{H_T}{3} - L_{key}\right) + P_{ah\omega} \cdot \left(\frac{H_T}{2} - L_{key}\right)\right]$ Moment from horizontal components (+ve)

$M_{av} := (P_{av} + P_{av\omega}) \cdot L_{foot}$ Moment from vertical components (-ve)

$M_\omega := P_\omega \cdot \left(L_{foot} - \frac{L_{heel}}{2}\right)$ Moment from surcharge above heel (-ve)

$M_I := (I_{stem} + I_{soil}) \cdot \left(\frac{H_w}{2} + L_{base}\right) + I_{foot} \cdot \frac{L_{base}}{2} - I_{key} \cdot \frac{L_{key}}{2}$ Moment from inertia forces (+ve)

The restoring moment from the self-weight of the wall and soil above the heel is calculated as follows without any load factor applied.

$$M_G := \left[W_{foot} \cdot \frac{L_{foot}}{2} + W_{stem} \cdot \left(L_{toe} + \frac{L_{stem}}{2}\right) + W_{key} \cdot \left(L_{foot} - \frac{L_{key}}{2}\right) + W_{soil} \cdot \left(L_{foot} - \frac{L_{heel}}{2}\right)\right]$$

$M_{ah} = 31.162 \cdot \frac{kN \cdot m}{m}$  $M_{av} = 39.264 \cdot \frac{kN \cdot m}{m}$  $M_I = 17.434 \cdot \frac{kN \cdot m}{m}$  $M_\omega = 4.05 \cdot \frac{kN \cdot m}{m}$

$M_G = 82.563 \cdot \frac{kN \cdot m}{m}$

$M_{net} := M_{ah} + M_I - M_{av} - M_\omega - M_G$  Net moment about toe, neglecting passive thrust

$M_{net} = -77.281 \cdot kN \cdot \frac{m}{m}$  Net moment must be < 0 for stability

$P_{vert} := W_{foot} + W_{stem} + W_{key} + W_{soil} + P_{av} + P_{av\omega} + P_\omega$

$P_{vert} = 94.03 \cdot \frac{kN}{m}$  Net vertical load on footing

$L_{net} := \frac{-M_{net}}{P_{vert}}$  $L_{net} = 0.822\,m$  Line of action of net vertical force (distance from toe)

$L_{third} := \frac{1}{3} \cdot L_{foot}$  $L_{third} = 0.617\,m$  $2 \cdot L_{third} = 1.233\,m$

Adjust wall proportions until line of action is within "middle third"

So the line of action of the net vertical force on the wall footing is still within the “middle third”.

**Check bearing capacity**

The "effective" width of the footing must be established, together with the net horizontal and vertical loads acting on the footing:

$B_{eff} := 2 \cdot L_{net}$  $B_{eff} = 1.644\,m < L_{foot} = 1.85\,m$  Effective footing width

$V_{ueq} := P_{vert}$  $V_{ueq} = 94.03 \cdot \frac{kN}{m}$  Ultimate vertical load on footing

$H_{ueq} := P_{ah} + P_{ah\omega} + I_{stem} + I_{soil} + I_{foot} + I_{key}$  Ultimate hozrizontal load on footing

$H_{ueq} = 50.722 \cdot \frac{kN}{m}$

Detailed bearing capacity calculations are appended, and give the following result:

$q_u = 223.059 \cdot \frac{kN}{m^2}$

$V_{ustar} := B \cdot q_u \cdot \Phi_{bc}$  $V_{ustar} = 366.651 \cdot \frac{kN}{m} > V_{ueq} = 94.03 \cdot \frac{kN}{m}$

$V_{star} > V_u$ therefore bearing capacity OK for earthquake case.

## Step 6. Wall sliding (earthquake case)

The sliding analysis is carried out with reference to the model shown in Figure X.3. The cohesive component of passive soil resistance in front of the toe of the wall was neglected because of possible desiccation and disturbance. None of the components of load or resistance are factored for the earthquake case.

**Check wall sliding on base**

$K_p := 1$ For $\phi = 0$

$P_p := 0.5 \cdot K_p \cdot \gamma \cdot (L_{base} + L_{key})^2 + 2 \cdot S_u \cdot L_{key}$ $P_p = 21.823 \cdot \frac{kN}{m}$ Ultimate passive resistance

$H_{star} := P_p + c_a \cdot B_{eff}$ $H_{star} = 104.01 \cdot \frac{kN}{m}$ Ultimate sliding resistance

$H_{star} = 104.01 \cdot \frac{kN}{m}$ $>$ $H_{ueq} = 50.722 \cdot \frac{kN}{m}$

$H_{star} > H_{ueq}$ therefore design OK

## Step 7. Wall stem bending strength (earthquake case)

The wall stem may fail in bending. The maximum bending moment will be at the base of the stem and may be calculated using the analytical model shown in Figure X.6. In this case the active earthquake pressure from the soil is added to the inertia of the wall stem. The calculation of the bending strength of the wall should be carried out in accordance with the relevant material code.

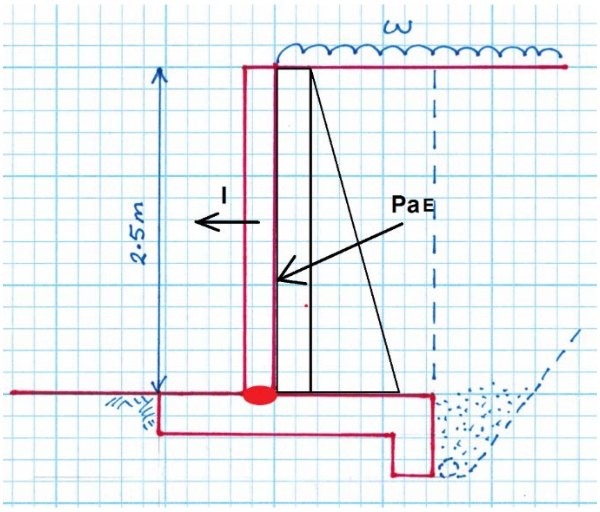



Figure X.6. Analytical model for calculating bending moment in wall stem (earthquake case)

**Calculate maximum bending moment in wall stem**

Assume that wall has water proof membrane with padding i.e. negligible interface friction

$\delta_s := 0$

$K_{aEs} := 0.473$ From M-O equations $\phi$ = 30 degrees, $\delta$ = 0, i = 0

$P_{as} := 0.5 \cdot K_{aE} \cdot \gamma \cdot H_w^2$ $P_{as} = 26.494 \cdot \frac{kN}{m}$ Active thrust

$P_{ahs} := P_{as} \cdot \cos(\delta_s)$ Horizontal component

$P_{a\omega s} := \omega_{eq} \cdot K_{aE} \cdot H_w$ $P_{a\omega s} = 4.71 \cdot \frac{kN}{m}$ Active thrust, surcharge component

$P_{ah\omega s} := P_{a\omega s} \cdot \cos(\delta_s)$ Horizontal component

$M_u := P_{ahs} \cdot \frac{H_w}{3} + P_{ah\omega s} \cdot \frac{H_w}{2} + I_{stem} \cdot \frac{H_w}{2}$ Ultimate bending moment in stem

$M_u = 31.028 \cdot kN \cdot \frac{m}{m}$

The bending capacity of the wall stem under action $M_u$ needs to be checked using the relevant material code.

**Detailed bearing capacity calculations:**

**Drained Bearing Capacity Shallow Footing - Vesic**

$B := B_{eff}$ $\quad L := 10 \cdot m$ $\quad D := L_{base}$ — Footing dimensions (effective)

$\beta := 0 \cdot deg$ — Ground slope in front of footing

$\eta := 0 \cdot deg$ — Tilt of footing (refer diagram)

$c := 0 \cdot \frac{kN}{m^2}$ — Soil effective cohesion

$c_a := 1.0 \cdot c$ — Adhesion (underside of footing)

$q := \gamma \cdot D$ — Surcharge

$$N_q := e^{\pi \cdot \tan(\phi)} \cdot \left(\tan\left(\frac{\phi}{2} + \frac{\pi}{4}\right)\right)^2 \quad N_c := \left(N_q - 1\right) \cdot \cot(\phi) \quad N_\gamma := 2 \cdot \left(N_q + 1\right) \cdot \tan(\phi)$$

$N_q = 18.401 \quad N_c = 30.14 \quad N_\gamma = 22.402$

Shape factors

$$\lambda_{cs} := 1 + \frac{B \cdot N_q}{L \cdot N_c} \quad \lambda_{\gamma s} := 1 - 0.4 \cdot \frac{B}{L} \quad \lambda_{qs} := 1 + \frac{B \cdot \tan(\phi)}{L}$$

$\lambda_{cs} = 1.112 \quad \lambda_{\gamma s} = 0.927 \quad \lambda_{qs} = 1.106$

Depth factors (D < B)

$$\lambda_{qd} := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot \frac{D}{B} \quad \lambda_{cd} := \lambda_{qd} - \frac{1 - \lambda_{qd}}{N_c \cdot \tan(\phi)}$$

$\lambda_{cd} = 1.042 \quad \lambda_{qd} = 1.039 \quad \lambda_{\gamma d} := 1$

Load inclination factors (loading parallel to B)

$$n_B := \frac{2 + \frac{B}{L}}{1 + \frac{B}{L}} \quad n_B = 1.845$$

$$\lambda_{qi} := \left(1 - \frac{H_u \cdot L}{V_u \cdot L + L \cdot B \cdot c_a \cdot \cot(\phi)}\right)^{n_B} \quad \lambda_{qi} = 0.327$$

$$\lambda_{\gamma i} := \left(1 - \frac{H_u}{V_u + L \cdot B \cdot c_a \cdot \cot(\phi)}\right)^{n_B + 1} \quad \lambda_{\gamma i} = 0.178$$

$$\lambda_{ci} := \lambda_{qi} - \frac{1 - \lambda_{qi}}{N_c \cdot \tan(\phi)} \quad \lambda_{ci} = 0.288$$

Ground inclination factors (see diagram)

$$\lambda_{qg} := (1 - \tan(\beta))^2 \quad \lambda_{cg} := \lambda_{qg} - \frac{1 - \lambda_{qg}}{N_c \cdot \tan(\phi)} \quad \lambda_{\gamma g} := \lambda_{qg}$$

$\lambda_{qg} = 1 \quad \lambda_{cg} = 1 \quad \lambda_{\gamma g} = 1$

Base tilt factors (see diagram)

$$\lambda_{qt} := (1 - \eta \cdot \tan(\phi))^2 \quad \lambda_{ct} := \lambda_{qt} - \frac{1 - \lambda_{qt}}{N_c \cdot \tan(\phi)} \quad \lambda_{\gamma t} := \lambda_{qt}$$

$\lambda_{qt} = 1 \quad \lambda_{ct} = 1 \quad \lambda_{\gamma t} = 1$

Ultimate bearing pressure

$$q_u := c \cdot \lambda_{cs} \cdot \lambda_{cd} \cdot \lambda_{ci} \cdot \lambda_{cg} \cdot \lambda_{ct} \cdot N_c + q \cdot \lambda_{qs} \cdot \lambda_{qd} \cdot \lambda_{qi} \cdot \lambda_{qg} \cdot \lambda_{qt} \cdot N_q + \frac{1}{2} \cdot \gamma \cdot B \cdot \lambda_{\gamma s} \cdot \lambda_{\gamma d} \cdot \lambda_{\gamma i} \cdot \lambda_{\gamma g} \cdot \lambda_{\gamma t} \cdot N_\gamma$$

$q_u = 91.968 \cdot \frac{kN}{m^2}$

$V_{ustar} := B_{eff} \cdot q_u \cdot \Phi_{bc} \qquad V_{ustar} = 84.082 \cdot \frac{kN}{m} \quad > \quad V_u = 79.787 \cdot \frac{kN}{m}$

**Undrained Bearing Capacity Shallow Footing - Vesic**

$S_u := 50 \cdot \frac{kN}{m^2}$  $\phi := 0$  $c := S_u$  $\gamma := 18 \cdot \frac{kN}{m^3}$  Soil parameters (undrained)

$B := B_{eff}$  $L := 10 \cdot m$  $D := L_{base}$  Footing dimensions (effective)

$\beta := 0 \cdot deg$  Ground slope in front of footing

$\eta := 0 \cdot deg$  Tilt of footing (refer diagram)

$\Phi_{bc} := 1.0$  Bearing capacity resistance factor

$c_a := 1.0 \cdot c$  Adhesion (underside of footing)

$q := \gamma \cdot D$  Surcharge

$N_c := 5.14$  $N_q := 1$  $N_\gamma := 0$  Bearing capacity factors

Shape factors

$\lambda_{cs} := 1 + \frac{B \cdot N_q}{L \cdot N_c}$  $\lambda_{\gamma s} := 1 - 0.4 \cdot \frac{B}{L}$  $\lambda_{qs} := 1 + \frac{B \cdot \tan(\phi)}{L}$  $\lambda_{cs} = 1.0$ $\lambda_{\gamma s} = 0.934$  $\lambda_{qs} = 1$

Depth factors

$\lambda_{qd} := 1 + 2 \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 \cdot \frac{D}{B}$  $\lambda_{cd} := 1 + 0.4 \cdot \frac{D}{B}$  $\lambda_{\gamma d} := 1$  $\lambda_{qd} = 1$  $\lambda_{cd} = 1.061$

Load inclination factors (loading parallel to B)

$n := \frac{2 + \frac{B}{L}}{1 + \frac{B}{L}}$  $\lambda_{ci} := 1 - \frac{n \cdot H_{ueq} \cdot L}{c_a \cdot N_c \cdot B \cdot L}$  $\lambda_{ci} = 0.777$  $\lambda_{\gamma i} := 1$  $\lambda_{qi} := 1$

Ground inclination factors

$\lambda_{cg} := 1 - \frac{2 \cdot \beta}{\pi + 2}$  $\lambda_{qg} := (1 - \tan(\beta))^2$  $\lambda_{\gamma g} := \lambda_{qg}$  $\lambda_{qg} = 1$  $\lambda_{cg} = 1$

$N_\gamma := -2 \cdot \sin(\beta)$  $N_\gamma = 0$

Base tilt factors

$\lambda_{ct} := 1 - \frac{2 \cdot \eta}{\pi + 2}$  $\lambda_{ct} = 1$  $\lambda_{\gamma t} := 1$  $\lambda_{qt} := 1$

Ultimate bearing pressure

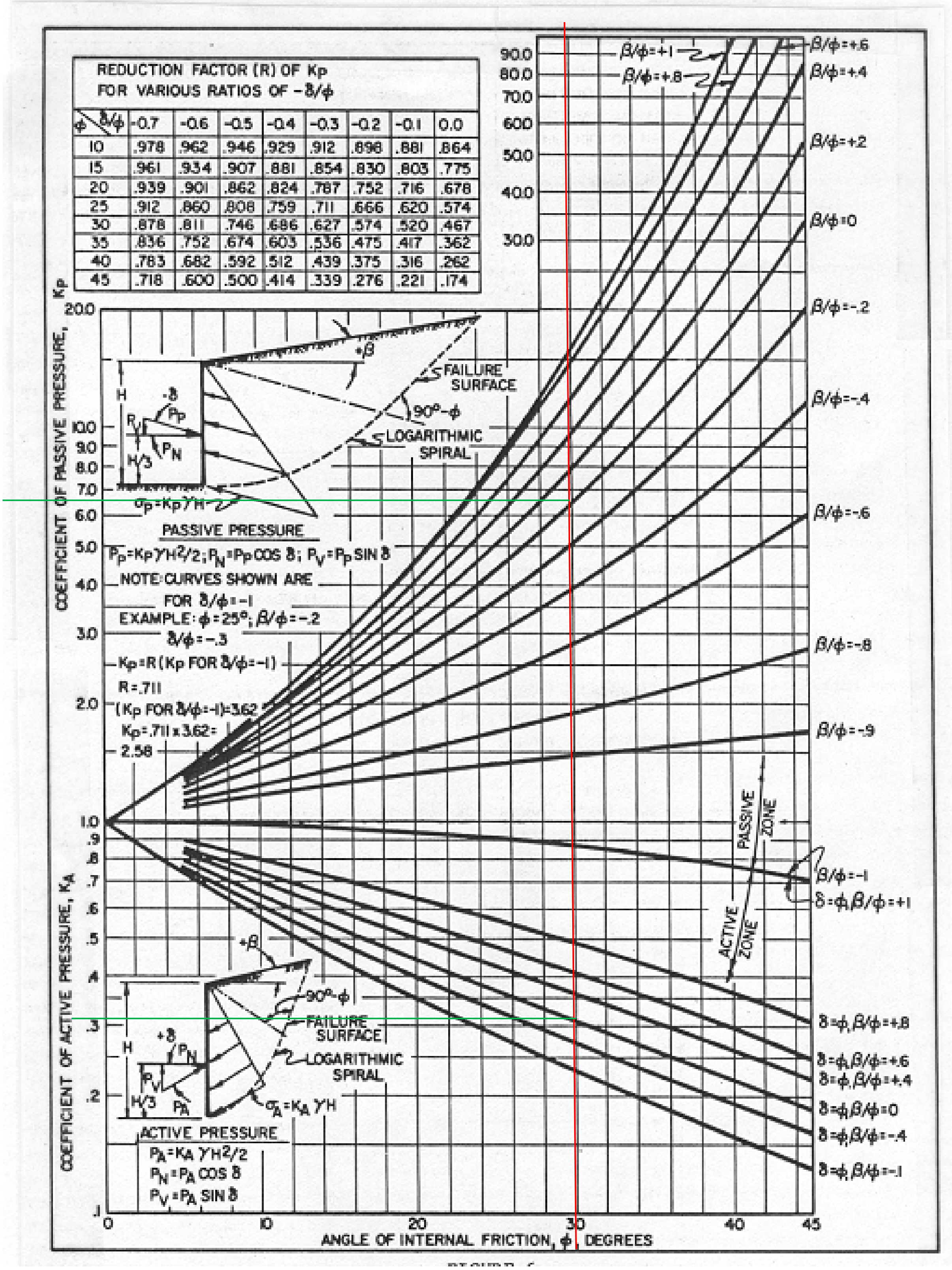
$q_u := c \cdot \lambda_{cs} \cdot \lambda_{cd} \cdot \lambda_{ci} \cdot \lambda_{cg} \cdot \lambda_{ct} \cdot N_c + q \cdot \lambda_{qs} \cdot \lambda_{qd} \cdot \lambda_{qi} \cdot \lambda_{qg} \cdot \lambda_{qt} \cdot N_q + \frac{1}{2} \cdot \gamma \cdot B \cdot \lambda_{\gamma s} \cdot \lambda_{\gamma d} \cdot \lambda_{\gamma i} \cdot \lambda_{\gamma g} \cdot \lambda_{\gamma t} \cdot N_\gamma$

$q_u = 223.059 \cdot \frac{kN}{m^2}$

$V_{ustar} := B \cdot q_u \cdot \Phi_{bc}$  $V_{ustar} = 366.651 \cdot \frac{kN}{m}$ > $V_{ueq} = 94.03 \cdot \frac{kN}{m}$

REDUCTION FACTOR (R) OF Kp FOR VARIOUS RATIOS OF $-\delta/\phi$

| $\phi$ \ $\delta/\phi$ | -0.7 | -0.6 | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.0 |
|---|---|---|---|---|---|---|---|---|
| 10 | .978 | .962 | .946 | .929 | .912 | .898 | .881 | .864 |
| 15 | .961 | .934 | .907 | .881 | .854 | .830 | .803 | .775 |
| 20 | .939 | .901 | .862 | .824 | .787 | .752 | .716 | .678 |
| 25 | .912 | .860 | .808 | .759 | .711 | .666 | .620 | .574 |
| 30 | .878 | .811 | .746 | .686 | .627 | .574 | .520 | .467 |
| 35 | .836 | .752 | .674 | .603 | .536 | .475 | .417 | .362 |
| 40 | .783 | .682 | .592 | .512 | .439 | .375 | .316 | .262 |
| 45 | .718 | .600 | .500 | .414 | .339 | .276 | .221 | .174 |



**References:**

Bowles, J.E. (1997) Foundation Analysis and Design, Fifth Edition, McGraw-Hill, New York, 1175 p.

Brinch-Hansen, J (1970). A revised and extended formula for bearing capacity, Bulletin No. 28, Danish Geotechnical Institute, Copenhagen.

Pender, M J (2015) "Moment and Shear Capacity of Shallow Foundations at Fixed Vertical Load". Proc., 12th Australia New Zealand Conference on Geomechanics, Wellington.

Vesic, A.S. (1975) Chap. 3, Foundation Engineering Handbook, 1st. Ed., edited by Winterkorn and Fang, Van Nostrand Reinhold, 751 p.